SPATIAL STABILITY OF A LIQUID FILM ON A ROTATING DISK

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One of the major assumptions made in the study of stability of viscous liquid layers is that of the parallel nature of the basic motion. This is true with reference to both boundary layers [1, 2] and thin liquid layers [2-4]. Such an assumption makes it possible to use basic flow parameters in the perturbed motion equations which are dependent on transverse coordinate only. At the present time there is much interest in the spatial nature of the flow in stability studies. This is true most of all with reference to boundary layers [5], where the main flow is weakly dependent on longitudinal coordinate (for a planar plate the parameters depend on $Re^{-1/2}x^{1/2}$). For a film on a rotating disk the velocity depends on $x^{1/3}$. Although the exponent of x is lower than in a boundary layer, we have no small parameter $Re^{-1/2}$ here, so that this is a more intense dependence. The present study will analyze spatial stability within the framework of linear theory. The asymptotic method developed in [6] will be used to obtain a solution.

We write the fundamental equations in the cylindrical coordinate system of [1]:

$$\partial u/\partial t + u\partial u/\partial r + v\partial u/\partial z - w^2/r = -\partial p/\rho\partial r + \nu(\partial^2 u/\partial r^2 + \partial u/r\partial r - u/r^2 + \partial^2 u/\partial z^2),$$

$$\partial v/\partial t + u\partial v/\partial r + v\partial v/\partial z = -\partial p/\rho\partial z + \nu(\partial^2 v/\partial r^2 + \partial v/r\partial r + \partial^2 v/\partial z^2),$$

$$\partial w/\partial t + u\partial w/\partial r + v\partial w/\partial z + uw/r = \nu(\partial^2 w/\partial r^2 + \partial w/r\partial r - w/r^2 + \partial^2 w/\partial z^2),$$

$$\partial u/\partial r + u/r + \partial v/\partial z = 0.$$

Here r lies in the radial plane and z is perpendicular thereto; u, v, and w are the radial, axial, and tangential velocity components; p is pressure; ν, kinematic viscosity; and ρ, density. Terms dependent on angle θ are omitted from these equations, since, in the future, we will assume that neither the basic nor perturbed flows are dependent thereon.

The boundary conditions for these equations are those on the disk and on the free surface

$$w = \Omega r, \quad u = 0, \quad v = 0 \quad (z = 0),$$
$$v = \partial a/\partial t + u\partial a/\partial r \quad (z = a),$$
$$p_{n\tau} = 0, \quad p_{nn} = -p_a + \frac{\sigma}{r}\frac{\partial}{\partial r}\left(r\frac{\partial a}{\partial r}\right),$$

where a is the film thickness; $p_{n\tau}$, stress tangent to the film surface; p_{nn}, normal stress; σ, surface tension; Ω, angular velocity. To study stability within the framework of the linear model, we represent all flow parameters in the form

$$u = W(u_y + u_\delta),\ v = W(v_y + v_\delta),\ w = W(w_y + w_\delta),\ a = a_0(y + \delta),$$

where W is the scale velocity; u_y, v_y, and w_y, corresponding components of the unperturbed velocity; u_δ, v_δ, and w_δ, corresponding perturbed-velocity components; a_0, linear scale factor; y, relative thickness of the unperturbed film; δ, amplitude of the surface perturbation, referenced to a_0.

We will carry the examination further using the variables

$$\xi = r/a_0, \quad n = z/a, \quad \tau = \tilde{W}t/a_0.$$

The problem of flow of a film on a rotating disk has been considered in [2, 4, 7-9]. In [2, 4, 7] asymptotic solutions were obtained, based on various approaches to the problem. In

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[8, 9] numerical calculations of the flow were carried out, with the solution of [8] considering the initial film profile. In the present case we will use the following asymptotic representation of the film parameters at large ξ:

$$u_y = \mathrm{Re}\left(n - \frac{1}{2}n^2\right)\xi^{-1/3} + \mathrm{Re}^3\left\{\frac{124}{315}\left(n - \frac{1}{2}n^2\right) - \frac{2}{9}\frac{\mathrm{Fr}^{-1}}{\mathrm{Re}^2}\left(n - \frac{1}{2}n^2\right) + \left(\frac{n^6}{630} - \frac{n^5}{60} - \frac{n^4}{36} + \frac{2}{9}n^3 - \frac{22}{45}n\right)\right\}\xi^{-9/3}, \quad (1)$$

$$w_y = \xi + \mathrm{Re}^2\left(\frac{n^3}{3} - \frac{n^4}{12} - \frac{2}{3}n\right)\xi^{-5/3},$$

$$y = \xi^{-2/3} + \mathrm{Re}^2\left(\frac{62}{315} - \frac{2}{9}\frac{\mathrm{Fr}^{-1}}{\mathrm{Re}^2}\right)\xi^{-10/3}, \quad \mathrm{Re} = \frac{a_0 W}{\nu}, \quad \mathrm{Fr}^{-1} = \frac{g a_0}{W^2}.$$

Analysis of these expressions and comparison to experimental and numerical results has shown [10] that Eq. (1) is usable up to $r/l \approx 1.5$, where $l = \left(\frac{9}{4\pi^2}\frac{Q^2}{\nu\Omega}\right)^{1/4}$ and Q is the volume flow rate. From the condition of conservation of mass and the boundary condition on the disk, we can define the scale factors

$$a_0 = \left(\frac{3}{2\pi}\frac{\nu Q}{\Omega^2}\right)^{1/5}, \quad W = a_0\Omega. \quad (2)$$

We will now consider the equations of the perturbed flow. Within the framework of linear stability theory, we write the equations of the perturbed flow and the boundary conditions:

$$\frac{\partial u_\delta}{\partial\tau} - \frac{\dot\delta}{y}n\frac{\partial u_y}{\partial n} + u_y\frac{\partial u_\delta}{\partial\xi} - \frac{\delta'}{y}u_y n\frac{\partial u_y}{\partial n} + \frac{v_\delta}{y}\frac{\partial u_y}{\partial n} - 2\frac{w_y w_\delta}{\xi} = \quad (3)$$

$$= -\frac{\partial p_\delta}{\rho\partial\xi} + n\frac{\delta'}{y}\frac{\partial p_y}{\rho\partial n} + \mathrm{Re}^{-1}\left\{\frac{\partial^2 u_\delta}{\partial\xi^2} - \frac{\delta''}{y}n\frac{\partial u_y}{\partial n} + 4\frac{y'\delta'}{y^2}n\frac{\partial u_y}{\partial n} - 2\frac{y'}{y}n\frac{\partial^2 u_\delta}{\partial\xi\partial n} - \right.$$

$$\left. - 2\frac{\delta'}{y}n\frac{\partial^2 u_y}{\partial\xi\partial n} + 2\frac{y'\delta'}{y^2}n^2\frac{\partial^2 u_y}{\partial n^2} + \xi^{-1}\frac{\partial u_\delta}{\partial\xi} + y^{-2}\left(\frac{\partial^2 u_\delta}{\partial n^2} - 2\frac{\delta}{y}\frac{\partial^2 u_y}{\partial n^2}\right)\right\},$$

$$\frac{\partial v_\delta}{\partial\tau} - \frac{\dot\delta}{y}n\frac{\partial v_y}{\partial n} + u_y\frac{\partial v_\delta}{\partial\xi} = -y^{-1}\left(\frac{\partial p_\delta}{\partial n} - \frac{\delta}{y}\frac{\partial p_y}{\partial n}\right) + \mathrm{Re}^{-1}\left\{\frac{\partial^2 v_\delta}{\partial\xi^2} + y^{-2}\left(\frac{\partial^2 v_\delta}{\partial n^2} - 2\frac{\delta}{y}\frac{\partial^2 v_y}{\partial n^2}\right)\right\} - \frac{\delta}{y}\mathrm{Fr}^{-1},$$

$$\frac{\partial w_\delta}{\partial\tau} + u_\delta\frac{\partial w_y}{\partial\xi} + \frac{w_y u_\delta}{\xi} = \mathrm{Re}^{-1}\left\{\frac{\partial^2 w_\delta}{\partial\xi^2} + y^{-2}\left(\frac{\partial^2 w_\delta}{\partial n^2} - 2\frac{\delta}{y}\frac{\partial^2 w_y}{\partial n^2}\right)\right\},$$

$$\frac{\partial u_\delta}{\partial\xi} - \frac{y'}{y}n\frac{\partial u_\delta}{\partial n} + \frac{y'}{y}n\frac{\delta}{y}\frac{\partial u_y}{\partial n} - \frac{\delta'}{y}n\frac{\partial u_y}{\partial n} + \xi^{-1}u_\delta + y^{-1}\frac{\partial v_\delta}{\partial n} - \frac{\delta}{y^2}\frac{\partial v_y}{\partial n} = 0;$$

$$u_\delta = 0, \quad v_\delta = 0, \quad w_\delta = 0 \ (n=0), \quad (4)$$

$$v_\delta = \dot\delta + u_y\delta' + y'u_\delta \quad (n = 1),$$

$$y^{-1}\frac{\partial u_\delta}{\partial n} + \frac{\partial v_\delta}{\partial\xi} - \frac{y'}{y}n\frac{\partial v_\delta}{\partial n} - \frac{\delta'}{y}n\frac{\partial v_y}{\partial n} = 2y'\left(\frac{\partial u_\delta}{\partial\xi} - y^{-1}\frac{\partial v_\delta}{\partial n}\right) + 2\delta'\left(\frac{\partial u_y}{\partial\xi} - y^{-1}\frac{\partial v_y}{\partial n}\right),$$

$$-p_\delta + 2\mathrm{Re}^{-1}\left(y^{-1}\frac{\partial v_\delta}{\partial n} - \frac{\delta}{y^2}\frac{\partial v_y}{\partial n}\right) = \left(\delta'' + \frac{\delta'}{\xi}\right)\mathrm{We}^{-1},$$

where $\dot\delta = \partial\delta/\partial\tau$; $\delta' = \partial\delta/\partial\xi$; $y' = dy/d\xi$; $\mathrm{We}^{-1} = \sigma/(\rho a_0 W^2)$.

It is evident from Eqs. (3) and (4) that at $\delta = 0$ we have a trivial solution of the system. As in spatial stability theory we will specify perturbations of the free surface in the form $\delta = e^{i\omega\tau}\bar\delta(\xi)$, where ω is the frequency of the oscillations, so that all defined parameters take on the form

$$\zeta_\delta = e^{i\omega\tau}\,\bar\zeta_\delta, \quad \zeta_\delta = \{u_\delta, v_\delta, w_\delta, p_\delta\}.$$

After substitution of these expressions in Eqs. (1) and (2), we obtain a problem of eigenvalues and eigenfunctions.

To find the solutions we represent the defined quantities as (we omit the bar above ζ_δ for simplicity)

$$\delta \sim \xi^m \exp\left[i(\gamma\xi^k + \ldots)\right], \quad u_\delta \sim \xi^p \exp\left[i(\gamma\xi^k + \ldots)\right],$$
$$v_\delta \sim \xi^r \exp\left[i(\gamma\xi^k + \ldots)\right], \quad p_\delta \sim \xi^s \exp\left[i(\gamma\xi^k + \ldots)\right]. \quad (5)$$

The terms of highest order in ξ are shown in Eq. (5). The definitions of m, p, r, s, and k are based on the requirement of regularity of the expansions and the condition that γ is an eigenvalue, i.e., $\gamma = \gamma(\omega)$. Maintaining the first three terms in the kinematic equation on the film surface, from the equation of conservation of mass and the last condition of Eq. (4) we find $m = r = p - {}^1/_3$, $s = p + {}^1/_3$, $k = {}^4/_3$. Thus, the solutions of Eq. (3) may be written in the form

$$\begin{aligned}
\delta &= A\xi^{p-1/3}\chi, \\
u_\delta &= A\xi^{p}(u_0 + \xi^{-2/3}u_1 + \xi^{-4/3}u_2 + \ldots)\chi, \\
v_\delta &= A\xi^{p-1/3}(v_0 + \xi^{-2/3}v_1 + \xi^{-4/3}v_2 + \ldots)\chi, \\
p_\delta &= A\xi^{p+1/3}(p_0 + \xi^{-2/3}p_1 + \xi^{-4/3}p_2 + \ldots)\chi, \\
w_\delta &= A\xi^{p-4/3}(w_0 + \xi^{-2/3}w_1 + \xi^{-4/3}w_2 + \ldots)\chi, \\
\chi &= \exp\,[i(\gamma_0\xi^{4/3} + \gamma_1\xi^{2/3} + \gamma_2\xi^{-2/3} + \ldots)],
\end{aligned} \tag{6}$$

where γ_j and p are wave numbers. To determine the flow stability it is now necessary to define γ_j and p from the equations and boundary conditions developed so far. Substituting Eq. (6) in Eq. (3) and boundary conditions (4), we obtain a chain of equations for determination of the eigenfunctions. Omitting the cumbersome intermediate steps, we will present the expressions obtained for p and γ_j:

$$\gamma_0 = -1{,}333\frac{\omega}{\mathrm{Re}}, \quad \gamma_1 = 3.556\gamma_0^3 + i1{,}580\gamma_0^4\mathrm{We}^{-1}, \tag{7}$$

$$p = -0{,}333 + \left(0{,}237 - 0{,}593\frac{\mathrm{Fr}^{-1}}{\mathrm{Re}^2}\right)\mathrm{Re}^2\gamma_0^2 - 9{,}739\gamma_0^6\mathrm{We}^{-1} + i\left(4.916\gamma_0^5 - 3.330\gamma_0^7\mathrm{We}^{-2}\right),$$

$$\gamma_2 = 0.724\gamma_0^3\mathrm{We}^{-1} - 0{,}439\gamma_0^5\mathrm{Re}^2\mathrm{We}^{-1} - 5{,}161\gamma_0^7 + 2.770\gamma_0^5\mathrm{Fr}^{-1}\mathrm{We}^{-1} +$$

$$+ 54.746\gamma_0^9\mathrm{We}^{-2} - i\left[3.160\left(0.038 + 1.067\frac{\mathrm{Fr}^{-1}}{\mathrm{Re}^2}\right)\gamma_0^4\mathrm{Re}^2 + 0.494\gamma_0^2 + 51{,}885\gamma_0^8\mathrm{We}^{-1} - 14.468\gamma_0^{10}\mathrm{We}^{-3}\right].$$

To determine the wave numbers (7) only the first terms of Eq. (1) were used. In order to include the second terms of Eq. (1) it is necessary to consider approximations which require a great deal of effort to determine, so, in order to consider the second terms of Eq. (1) only those portions of γ_3 and γ_4 were found which are directly related to these terms. As a result, we have the following equations to complement Eq. (7):

$$\gamma_3 = -1.282\,\mathrm{Re}^2\gamma_0, \quad \gamma_4 = -2{,}339\,\mathrm{Re}^2\gamma_0^3 - i\,3.320\gamma_0^4\mathrm{Re}^2\mathrm{We}^{-1}.$$

Thus, the propagation of surface perturbations can be described by the equations

$$\frac{\delta}{y} = \exp\left[i(\omega\tau + f_i) + f_r\right], \tag{8}$$

$$f_i = \gamma_0\xi^{4/3} + 3{,}556\gamma_0^3\xi^{2/3} + (4{,}916\gamma_0^5 - 3.330\gamma_0^7\mathrm{We}^{-1})\ln\xi + \left[0.724\gamma_0^3\mathrm{We}^{-1} - 1{,}778\left(0{,}247 - 1.580\frac{\mathrm{Fr}^{-1}}{\mathrm{Re}^2}\right)\gamma_0^5\mathrm{Re}^2\mathrm{We}^{-1} -\right.$$

$$\left. - 5.161\gamma_0^7 + 54.746\gamma_0^9\mathrm{We}^{-2}\right]\xi^{-2/3} - 1{,}282\,\mathrm{Re}^2\gamma_0\xi^{-4/3} - 2.339\,\mathrm{Re}^2\gamma_0^3\xi^{-6/3},$$

$$f_r = -1{,}580\gamma_0^4\mathrm{We}^{-1}\xi^{2/3} + \left[1.778\left(0.133 - 0.333\frac{\mathrm{Fr}^{-1}}{\mathrm{Re}^2}\right)\gamma_0^2\mathrm{Re}^2 - 9{,}739\gamma_0^6\mathrm{We}^{-1}\right]\ln\xi + \left[3.160\left(0{,}038 + 1.067\frac{\mathrm{Fr}^{-1}}{\mathrm{Re}^2}\right)\times\right.$$

$$\left.\gamma_0^4\mathrm{Re}^2 + 0{,}494\gamma_0^2 + 51{,}885\gamma_0^8\mathrm{We}^{-1} - 14.468\gamma_0^{10}\mathrm{We}^{-3}\right]\xi^{-2/3} + 3.320\gamma_0^4\mathrm{Re}^2\mathrm{We}^{-1}\gamma^{-6/3}.$$

It follows from Eq. (8) that perturbations can propagate only in the direction of increasing ξ, which has been confirmed by experiment [10]. If we now differentiate the imaginary component of the exponent with respect to τ, we obtain the dimensionless phase velocity

$$c_\Phi = -\omega\left\{1{,}333\gamma_0\mathrm{We}^{-1}\xi^{1/3} + 2{,}370\gamma_0^3\xi^{-1/3} + (4.916\gamma_0^5\right.$$

$$- 3{,}330\gamma_0^7\mathrm{We}^{-2})\xi^{-1} - 0{,}667\left[0.724\gamma_0^3\mathrm{We}^{-1} - \left(0{,}247 - 1.580\frac{\mathrm{Fr}^{-1}}{\mathrm{Re}^2}\right)\times\right.$$

$$\left.\left.\times 1.778\gamma_0^5\mathrm{Re}^2\mathrm{We}^{-1} - 5.161\gamma_0^7 + 54{,}746\gamma_0^9\mathrm{We}^{-2}\right]\xi^{-5/3} + 1.709\,\mathrm{Re}^2\gamma_0\xi^{-7/3} + 4{,}678\gamma_0^3\mathrm{Re}^2\xi^{-9/3}\right\}^{-1},$$

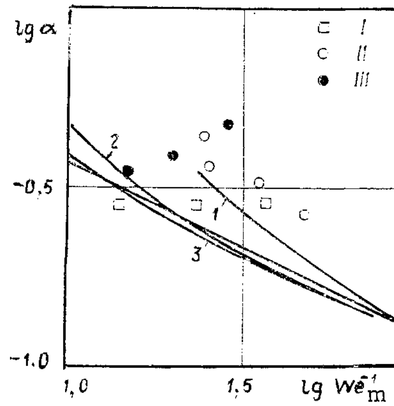

Fig. 1

whence it is evident that c_ϕ has a minimum at some γ_0. The function f_r indicates the change in perturbation amplitude. At $f_r > 0$ the perturbations increase with increase in ξ, while for $f_r < 0$ they decrease. For the case $Re^2 < (5/2)Fr^{-1}$, f_r decreases with increasing ξ for all wavelengths, i.e., in this case the film flow is absolutely stable. For $Re^2 > (5/2)Fr^{-1}$ the quantity f_r depends on wavelength. When short-wavelength perturbations develop, as they propagate at some distance ξ the first term in f_r begins to have an effect and these perturbations damp out. The smaller γ_0, the greater the distance from the center of the disk to which the corresponding perturbations propagate. An increase in perturbations leads to the appearance of waves on the film surface. Wave motion of a film on a disk was studied experimentally in [10-12]. It was shown in [12] that waves on the film surface appear at some distance

$$L_{in} = (Q/(2\pi\nu^{1/2}\Omega^{1/2}))^{1/2}.$$

The further behavior of the waves depends on the flow rate supplied and the angular velocity of the disk. It was demonstrated numerically in [9] that at some liquid flow rate "flooding" of the liquid sets in. The maximum flow rate at which "flooding" of the liquid sets in at a distance equal to the disk radius is given by the expression

$$Q = 1.9\pi R_\delta^2\Omega^{1/2}\nu^{1/2}.$$

It is interesting that if we replace the disk radius by the current radius corresponding to the amount of "flooding," we obtain a value close to L_{in} of [12]:

$$R_3 = (Q/(1.9\pi\nu^{1/2}\Omega^{1/2}))^{1/2}.$$

In the present case we cannot describe the development and propagation of perturbations in this region since $L_{in} < l$, and the asymptotic expansion of Eq. (1), as was indicated above, is valid for $r/l \gtrsim 1.5$. However, with the aid of Eq. (8) we can obtain the characteristics of surface waves, in particular the wave number $\alpha = 2\pi\delta/\lambda$. As is evident from the last expression of Eq. (8), f_r has a maximum at some value γ_0. This corresponds to the most rapidly increasing perturbations. Figure 1, taken from [10], shows curves of wave numbers calculated for the following perturbations: curve 1) Re = 47.537, We^{-1} = 88.868; 2) Re = 65.560, We^{-1} = 24.542; 3) Re = 75.303, We^{-1} = 14.096; the points I are for $\nu^* = 1$, $\sigma^* = 0.83$; II) $\nu^* = 1$, $\sigma^* = 0.96$; III) $\nu^* = 2.6$, $\sigma^* = 0.85$ (ν^* and σ^* are the kinematic viscosity and liquid surface tension coefficients referenced to the corresponding quantities for distilled water).

The straight line corresponds to the most rapidly increasing perturbations in films on a vertical surface. The abscissa indicates local Weber numbers $We^{-1}_m = 9We^{-1}Re^{-2}\xi^{4/3}$. It is evident from Fig. 1 that wave numbers decrease with increase in Weber number. At constant Re and We^{-1} increase in the local Weber number We^{-1}_m is related to increase in ξ, while increase in ξ causes the problem of film flow on the disk to coincide with that of film flow on a wall [2]. Hence it follows that with increase in ξ the wave numbers approach those of a film flowing on a wall.

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SPATIAL DISTORTION OF MEAN BOUNDARY LAYER BY NATURAL OSCILLATIONS

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1. One of the stages in the nonlinear growth of disturbances in the transition region of an incompressible boundary layer on a flat plate is the generation and subsequent growth of three-dimensional oscillating field as a result of which disturbances are found to have clearly defined spatial structure with alternating maxima (crests or peaks) and minima (valleys) of amplitudes in the transverse direction (along z axis). Reasons for the appearance of such natural wave structure have not yet been explained conclusively. One of them could be the interaction of the initial finite amplitude plane disturbances with small, local, spatial nonuniformities in the mean flow, which leads to the generation of a pair of oblique Tollmien—Schlichting waves [1]. Natural weak disturbances in the leading edge region can also be the source of subsequent real wave fields.

Subsequent triple-wave resonant interaction in the nonlinear growth region of plane waves lead to the amplification of three-dimensional components [2, 3]. Thus, it was shown in [4] that on attaining the threshold amplitudes $\varkappa_d \sim 0.007$ there is a strong growth of oblique waves so that the characteristic disturbance field of the boundary layer takes the form of an additive field of Tollmien—Schlichting waves:

$$\begin{aligned} u'(x, y, z, t) &= \varkappa_d u_d(y) e^{\Omega_1} + 2\varkappa_т u_т(y) e^{\Omega_2} \cos\beta z, \\ v'(x, y, z, t) &= \varkappa_d v_d(y) e^{\Omega_1} + 2\varkappa_т v_т(y) e^{\Omega_2} \cos\beta z, \\ w'(x, y, z, t) &= 2\varkappa_т i w_т(y) e^{\Omega_2} \sin\beta z, \end{aligned} \quad (1.1)$$

where $\Omega_1 = i\alpha_1(x - C_1 t)$, $\Omega_2 = i\alpha_2(x - C_2 t)$. The inclination of oblique waves to the plane flow is determined as $\theta = \arctan \beta/\alpha$. The presence of such characteristic disturbances leads to a qualitative change in the structure of the mean flow, viz., minimum values of mean velocity at crests and maximum values at valleys are observed. This is interpreted as the appearance of a system of localized streamwise vortices in the boundary layer which are periodic

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